Psychometric Theory

Homework 3: Regression answers

Problems

Assume

	Verbal	Quant	GPA
Verbal	1.0		
Quant	.6	1.0	
GPA	.3	.2	1.0
Mean	500	550	3.0
Sigma	120	100	.5

1) For a student with a GRE Verbal of 700, what is her expected GRE Quantscore?
2) For a student with a GRE Quant of 750, what is the expected GPA?
3) What is the correlation of Verbal with GPA holding Quant constant?
4) What is the multiple correlation of Verbal + Quant with GPA?
5) For a student with a Verbal of 700 and a Quant of 700, what is the expected GPA?

Simple prediction

1) For a student with a GRE Verbal of 700, what is her expected GRE Quantscore?

$$z_{ypredicted} = r_{xy}\, z_x$$

$$z_{\text{ quant predicted}} = .6 *(700\text{-}500)/120 = 1.0$$

$$Q_{\text{uant predicted}} = z_{\text{ quant predicted}} * sd_{\text{ quant}} + mean_{quant}$$

$$Q_{\text{uant predicted}} = 1.0 * 100 + 550 = 650$$

simple prediction

2) For a student with a GRE Quant of 750, what is the expected GPA?

$$z_{ypredicted} = r_{xy}\, z_x$$

$$z_{\text{gpa predicted}} = .2 * (750-550)/100 = .4$$

$$GPA_{\text{predicted}} = z_{\text{gpa predicted}} * sd_{\text{gpa}} + mean_{gpa}$$

$$GPA_{\text{predicted}} = .4 * .5 + 3.0 = 3.2$$

Partial correlation

- 3) What is the correlation of Verbal with GPA holding Quant constant?
- partial $r_{xy.z}$ =
  - $(r_{xy} - r_{xz}*r_{yx})/sqrt((1-r_{xz}^2)*(1-r_{yz}^2))$
  - $(.3 - .2*.6)/sqrt((1-.6^2)*(1-.2^2)) = .2296397$

Multiple R

- 4) What is the multiple correlation of Verbal + Quant with GPA?
- $r_{\text{GRE V, gpa}} = .3$ $r_{\text{GRE Q, gpa}} = .2$ $r_{\text{GRE V,Q}} = .6$
- beta $y.x = (r_{xy} - r_{xz} * r_{yz}) / (1 - r_{xz}^2)$
- beta $_{\text{GRE V, gpa}} = (.3 - .6 * .2) / (1 - .6^2) = .28125$
- beta $_{\text{GRE Q, gpa}} = (.2 - .6 * .3) / (1 - .6^2) = .03125$
- $R^2 = \text{beta}_{y.x} * r_{xy} + \text{beta}_{y.z} * r_{yz} \ldots$
- $R^2 = \text{beta}_{\text{GRE Q, gpa}} * r_{\text{GRE Q, gpa}} + \text{beta}_{\text{GRE V, gpa}} * r_{\text{GRE V, gpa}}$ =
- $R^2 = .28125 * .3 + .03125 * .2 = .090625$
- $R = .3010399$

Multiple correlation: prediction

- 5) For a student with a Verbal of 700 and a Quant of 700, what is the expectedGPA?
- Note that the betas found in problem 4 were standardized betas. So, in this case
- $z_{ypredicted|(x1,x2)} = beta_{x1} * z_{x1} + beta_{x2} * z_{x2}$
- $r_{GRE\ V,\ gpa} = .3$ $r_{GRE\ Q,\ gpa} = .2$ $r_{GRE\ V,Q} = .6$
- $beta\ y.x = (r_{xy} - r_{xz} * r_{yz}) / (1 - r_{xz}^2)$
- $beta_{GRE\ V,\ gpa} = (.3 - .6 * .2)/(1 - .6^2) = .28125$
- $beta_{GRE\ Q,\ gpa} = (.2 - .6 * .3)/(1 - .6^2) = .03125$
- z gpa predicted given verbal and quant =
  - .28*(700-500)/120 + .03 * (700-550)/100 = .515625
- $GPA_{predicted} = z_{gpa\ predicted} * sd_{gpa} + mean_{gpa} =$
  - .515625*.5+ 3.0 = 3.258